OC Problem Set 4

Friday, May 22, 2015.

Problem 1: Slab Waveguide

A symmetric slab waveguide as depicted in Figure 2.7 in the lecture notes consists of three homogenous, isotropic layers, which are infinitely extended along the *y* and z axes. (thus $\partial/\partial y = 0$ in the following). The core area with height h and refractive index n_1 is surrounded by the infinitely extended cladding medium with refractive index $n_2 < n_1$.

a) Explain the following terms with the help of Figures 2.7 and 2.8 in the lecture notes: Total internal reflection (TIR), phase difference of reflected and incident wave after TIR, evanescent field, standing wave, *E*-polarized (TE-wave or H-wave), *H*-polarized (TM-wave or E-wave), modes of a slab waveguide, symmetric/antisymmetric modes.

b) An important property of a guided mode is that its field distribution does not change while propagating, i.e.

$$E_{x,y,z}(x,z,t) = E_{x,y,z}(x)\exp\left(\mathrm{j}(\omega t - \beta z)\right).$$

In the following, we calculate the eigenvalue equation for the case of *symmetric* TE-modes of the slab waveguide. As an ansatz for $\vec{E}(x)$ we choose (Eq. 2.44, 2.45)

$$E_y(x) = \begin{cases} B\exp\left(-|k_{2x}|x\right) & \text{for } \mathrm{x} > h/2 \\ A\cos\left(k_{1x}x\right) & \text{for } |x| < h/2 \\ B\exp\left(|k_{2x}|x\right) & \text{for } x < -h/2 \end{cases} \quad \text{and } E_x = E_z = 0.$$

Before proceeding with the calculation: Which relationship holds between k_{1x} and β, which relation exists between k_{2x} and β, respectively?

Using Maxwell's equations, calculate the *z*-component of the magnetic field $\vec{H}$ from the electric field ansatz. Apply the amplitude boundary condition for the electric and magnetic fields at $|x| = h/2$ and derive the eigenvalue equation 2.41 by eliminating *A*. What changes, if *antisymmetric* modes are considered? What changes, if we assume TM-modes instead of TE-modes?

c) Now assume $h = 10\ \mu\mathrm{m}$, $\lambda = 1.5\ \mu\mathrm{m}$, $n_1 = 1.45$ and $n_2 = 1.44$. Solve the eigenvalue equation graphically, by plotting both the right hand and the left hand side of equation 2.41 as a function of u_{pm} in the same diagram, and by evaluating the intersection points. How many modes can propagate in the given case? What is the minimum number of modes that can always propagate in a symmetric slab waveguide?

d) With the results from c), determine for all propagating modes the propagation constant β, the penetration depth $1/|k_{2x}|$ of the field into the cladding medium, and the angle ϑ between the k_1-vector and the *z*-axis in the core. What is the associated incident angle that corresponds to the angle ϑ?

Problem 2: Evanescent Wave

The evanescent wave in the upper cladding material ($x > h/2$) of a slab waveguide is given by

$$E_{2y}(x) = B\exp(-|k_{2x}|x)\cos(\omega t - k_{2z}z) \ .$$

a) Sketch the evanescent field $E_{2y}(x)$ at the plane $z = 0$ for the time instances $t = 0$, $t = \pi/(2\omega)$ and $t = \pi/\omega$.

b) In which direction is the evanescent wave propagating?

c) Show that there is no energy flux in x-direction.

   Hint: Calculate the Poynting vector of the evanescent wave and integrate over a period of $T = 2\pi/\omega$.

For questions and suggestions on the OC tutorial please contact:

Stefan Wolf, Bldg. 30.10, Room 1.23, Phone: 0721/608-47173, s.wolf@kit.edu
Wladick Hartman, Room 2.23, Phone: 0721/608-48954, wladislaw.hartmann@kit.edu